

On maximum loss of information of any odd-fractional design in 2^k factorial experiment

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ABSTRACT

A shorter procedure of determining maximum loss of information in odd fractional 2^k factorial experiment has been developed. It serves as a quick check on the confounded effects so that all information on treatment effects will not be lost.

INTRODUCTION

When the number of factors is large or when only a few of the experimental effects are of interest then it is not practicable or necessary to carry out a complete 2^k factorial experiment. Factorial experiments provide a means of estimating those experimental effects which are of interest without carrying out a complete experiment.

Factorial experiment often leads to great economy in experimentation particularly if the runs can be made sequentially Montgomery (1976). This can be done using a design technique called confounding. Confounding is a design technique for arranging a complete factorial experiment in blocks where the block size is smaller than the number of treatment combinations in one replicate Montgomery (1976), Raghavarao (1971). As a result of confounding treatment effects in blocks, information on the confounding effects is lost, where there is complete confounding, total loss information is accounted for the treatment effect. And where there is partial confounding, partial loss of information is accounted for the treatment effect.

Loss of information can be defined as (1-relative efficiency or information) Pazman (1986), Yate (1973), Chigbu (1998), John (1987).

But Onokogu (1997) defines loss of information as:

Number of degrees of freedom (of the interactions) confounded

Number of degrees of freedom (of such interactions) available

Kemphorne (1952), Naira and Rao (1942) mention a remarkable property that, if in any design, every treatment is replicated the same number of times, the total relative loss of information is one less than the average number of blocks per replication.

Theorem:

Given an odd-fractional design ξ in 2^k

Factorial experiment where N is the number of experimental effects (treatments) to be secured by scarifying the highest interaction in the

experiment. The maximum loss of information for the design ξ_N is given as $\left(\frac{2^k}{N}\right)-1$, where $K < N < 2^k - 1$

Proof:

Loss of information is defined as

Number of degrees of freedom (of the interactions) confounded

Number of degrees of freedom (of such interactions) available

(Onokogu, 1997).

where, the number of degree of freedom (of interaction) confounded is equal to the number of blocks degree of freedom

Let N be the size of ξ_N ,

It implies that the number of blocks needed to confound N items i.e.

experimental treatment effects is $\frac{2^k}{N}$ blocks.

Hence, the number of blocks degree of freedom = $\frac{2^k}{N} - 1$ which is number of degree of freedom (of interaction) confounded.

But loss of information =

$$\frac{2^k}{N} - 1$$

Number of degrees of freedom (of such interactions) available

The number of degree of freedom (of such interaction) available is given as:

$$n_a = \sum_{j=2}^n nj(a-1)^{j-1} \quad \text{Onokogu (1997)}$$

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Manuscript received by the Editor May 3, 2005; revised manuscript accepted July 21, 2006.

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Here $a=2$ (level of the experimental factors) confounding the highest order of interactive effect with the blocks, which is customarily sacrificed and is the only available factor interaction that must be confounded.

$\Rightarrow n_a = 1; n_j = 1$ (number of highest order of interactive effect). where Π_j is the number of j factors interactions that must be confounded, K is the factor in the experiment and n is the number of interactive terms in the experiment.

Hence maximum loss information is
$$\frac{\left(\frac{2^K}{n}-1\right)}{1} = \frac{2^K}{N}-1$$

IMPLEMENTATION

Given $\xi_N = \frac{5}{8} \times 2^3$

where $r = 5$ and $k = 3$

The total loss of information

$$L.I = \frac{2^3}{5}-1 = \frac{8}{5}-1 = \frac{13}{5}-1 = \frac{3}{5}$$

CONCLUSION

From the foregoing, the loss of information of odd-fractional experiments in $2K$ factorial experiment is given as

$$\frac{2^K}{N}-1$$

This is for any odd-fraction. The examples used for verification were chosen because we can easily illustrate how to get the total loss of information with ease without much computation.

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